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OCCUPATIONAL MATHEMATICS; EQUIVALENT FORMS. REPORT NO. 16-C. BOOKLET II. FINAL REPORT.

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This programed mathematics textbook is for student use in vocational education courses. It was developed as part of a programed series covering 21 mathematical competencies which were identified by university researchers through task analysis of several occupational clusters. The development of a sequential content structure was also based on these mathematics competencies. After completion of this program the student should be able to: (1) change integers into equivalent forms, (2) change fractions into equivalent forms, (3) recognize prime numbers up to 20, (4) factor the number 100 into primes, and (5) reduce literal or numeric fractions. The material is to be used by individual students under teacher supervision. Twenty-six other programed texts and an introductory volume are available as VT 006 882-VT 006 882-VT 006 909, and VT 006 975. (EM)

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Occupational Mathematics

EQUIVALENT FORMS

June 1968

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Occupational Mathematics

EQUIVALENT FORMS

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Report No. 16-C

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Washington State University, Department of Education, Pullman, Washington
State Coordinating Council for Occupational Education, Olympia, Washington

OBJECTIVES

1. The student should be able to change integers into equivalent forms.

2. The student should be able to change fractions into equivalent forms.

3. The student should be able to recognize prime numbers up to 20.

4. The student should be able to factor a number 100 into primes.

5. The student should be able to reduce literal or numeric fractions.

Greetings! You are about to begin improving your knowledge of basic mathematics. There are many important uses for the mathematics you are learning.

This booklet is not like your ordinary books. It is designed to help you learn as an individual. On the following pages you will find some information about mathematics. After the information is presented, you will be asked a question. Your answers to these questions will determine how you proceed through this booklet. When you have selected your answer to the question, turn to the page you are told to.

Do not write in this booklet. You may wish to have a pencil and some paper handy so you can write when you want to.

Remember this is not an ordinary book.

1. Study the material on the page.
2. Read the question on the page (you may want to restudy the material on the page).
3. Select the answer you believe is correct.
4. Turn to the page indicated by your answer.

Are you ready to begin?

(a) Yes — Turn to page 1

(b) No — Turn to page C

(c) HELP — Go see your teacher

Your answer was (b) No.

Well, this booklet is a little different.

Go back and read page B again. After you have read it you will probably be ready to begin.

You are already familiar with numbers such as 1, 2, 3, 4 and 5. These numbers are called integers. Integers may be expressed in fractional forms. For example, the integer 2 can be expressed as the fraction 6/3. 2/1 and 6/3 are called equivalent forms of 2 since they represent the same quantity.

Which of the following is an equivalent form of the integer 4?

(a) 8/4 Turn to page 2

(b) 4/8 Turn to page 2

(c) 8/2 Turn to page 12

Your answer was (a) 8/4 or (b) 4/8. Incorrect.

The correct answer was 8/2.

8/2 is the equivalent form of 4 as 8/2 = 4.

Let's try another one.

The fraction 10/5 is an equivalent form of _____?

(a) 10 Turn to page 5

(b) 2 Turn to page 7

(c) 5 Turn to page 5

Your answer was (a) 3. Very good!

You saw that if a = 9 and 6 = 3, then a/b = 9/3 = 3. Therefore, we can say that a/b, for the values given, is an equivalent form of 3.

Now turn to page 20 and we'll find out why equivalent forms are important. Meet you there.

Your answer of (b) 9/3, 3/1 is correct.

You apparently understand that we can write equivalent forms for integers and that more than one is correct.

We can also write equivalent forms using letters. Letters just represent numbers so they can be handled the same way. For example, 8/8 = 1. If we used a letter, K, for 8 then we have K/K = 1. K/K is also an equivalent form of the integer 1.

Which of the following is a correct equivalent form?

(a) a/b = 1 Turn to page 6

(b) c/c = 1 Turn to page 9

Your last answer was incorrect. You seem to be having trouble understanding equivalent forms.

Ask your human teacher for help and then return to page 1.

Your answer was (a) a/b = 1. Incorrect!

Remember that letters just represent integers.

a/a, b/b, c/c (a, b, c not zero) are all equivalent forms of 1.

Are 8/8 and x/x (x not zero) equivalent forms of 1?

(a) Yes Turn to page 14

(b) No Turn to page 16

Your answer was (b) 2. Very good! Now you're on the right track.

You recognized that the fraction 10/5 was an equivalent form of the number 2.

An integer may have more than one equivalent form. For example, 8/4, 6/3, 10/5, 4/2, 2/1 are equivalent forms of 2.

Which of the following are equivalent forms of the integer 3?

(a)	6/3, 8/3	Turn to page 8
(b)	9/3, 3/1	Turn to page 4
(c)	Both a and b	Turn to page 10
(d)	Don't understand	Turn to page 8

Incorrect!

Let's take another look at what equivalent forms are. For example, we want to see which fractions are equivalent to the integer 2. We see that the following fractions are equivalent as they reduce to 2 by dividing the bottom term into the top one.

10/5, 8/4, 6/3, 2/1 all equal 2.

Let's try another one.

Which of the following is an equivalent form to the relations 8/2, 4/1?

(a) 4 Turn to page 11

(b) 2 Turn to page 5

(c) 8 Turn to page 5

Your answer was (b) f/f = 1 or (c) c/c = 1. Good!
You are catching on.

Now let's consider the case where the letters take on numerical values.

If a = 9 and b = 3, then a/b is an equivalent form for _____?

(a)	3	Turn to page 3
(b)	9	Turn to page 17
(c)	1	Turn to page 13
(d)	I don't understand	Turn to page 13

Your answer was (c) Both a and b. Incorrect!

We were looking for equivalent forms of the integer 3.
It is correct that 9/3, 3/1 are equivalent to 3 but
6/3 = 2 is not 3.

Made a careless mistake, huh?
Let's look at another one a little more carefully.

Which of the following is an equivalent form to the relations 8/2, 4/1?

(a) 4 Turn to page 11

(b) 2 Turn to page 5

(c) 8 Turn to page 5

Your answer was (a) 4. Correct!

You apparently understand that we can write equivalent forms for integers and that more than one is correct.

Let's try one more.

Are the forms 8/8 and K/K (K not zero) equivalent forms of 1?

(a) Yes Turn to page 14

(b) No Turn to page 16

Your answer was 8/2. Good!

You saw that the equivalent form of the fraction 8/2 is 4, since 8/2 = 4.

An integer may have more than one equivalent form. For example, 8/4 = 2, but 6/3 = 2, 10/5 = 2 4/2 = 2, 2/1 = 2 are also true.

Which of the following are equivalent forms of the integer 3?

(a)	6/3, 8/3	Turn to page 8
(b)	9/3, 3/1	Turn to page 4
(c)	Both a and b	Turn to page 10
(d)	I don't understand	Turn to page 8

Incorrect!

If a = 9 and b = 3, then a/b = 9/3 which is equivalent to 3.

Let's try another one.

The integer 4 can be expressed by which of the following equivalent forms?

(a) 4/2 Turn to page 5

(b) 8/2 Turn to page 15

(c) 4/1 Turn to page 15

Your answer was (a) Yes. Correct!

8/8 is equivalent to 1 and K/K = 1.

You know that we can write equivalent forms using letters. Letters just represent numbers so they can be handled the same way.

Which of the following is a correct equivalent form?

(a) a/b = 1 Turn to page 6

(b) c/c = 1 Turn to page 9

(c) f/f = 1 Turn to page 9

Your answer was (b) 8/2 or (c) 4/1. Very good!

Now you've got it.

Here's another one just like it.

What integer is equivalent to the forms 20/2, 40/4?

(a) 10 Turn to page 18

(b) 2 Turn to page 5

(c) 4 Turn to page 5

Your answer was (b) No. Incorrect!

You seem to be having trouble recognizing equivalent forms.

Tell your human teacher where you are and then return to the beginning of this section.

Incorrect!

If a = 9 and b = 3, then a/b = 9/3 which is equivalent to 3.

Let's try another one.

The integer 4 can be expressed by which of the following equivalent forms?

(a) 4/2 Turn to page 5

(b) 8/2 Turn to page 15

(c) 4/1 Turn to page 15

Your answer was (a) 10. Correct! You are on your way.

Now that you understand what an equivalent form is for integers and that letters are handled the same way, turn to page 20 and we'll find out why equivalent forms are important.

Your answer was incorrect.

You are having your ups and downs. Return to the beginning of this section, page 20, and carefully follow the examples and do the problems again.

Good! Let's continue.

Suppose we are confronted with a problem such as

$\frac{3 \times 2}{2}$ and were asked to reduce it.

The method we use is to apply our equivalent form idea.

First we see that $\frac{3 \times 2}{2}$ can be separated into $3 \times \frac{2}{2}$.

Now applying our equivalent form, we have $\frac{2}{2} = 1$.

Therefore, by substituting 1 for $\frac{2}{2}$ we have $3 \times 1 = 3$.

Pretty slick, huh?

Now you try one.

If we want to reduce $\frac{4 \times 3}{3}$ we should first:

(a) Change it to $4/3 \times 3$ Turn to page 24

(b) Change it to $4 \times 3/3$ Turn to page 26

(c) Not sure Turn to page 24

Your answer was incorrect!

You seem to be having trouble recognizing equivalent forms. Equivalent forms are necessary to reduce fractions.

Therefore, return to page 1 and review equivalent forms.

Oops! You probably made a careless mistake.

Try this one.

We want to reduce the fraction $\frac{4 \times 9}{9}$. The simplified form will look like:

(a) $4 \times 9 = 36$ Turn to page 21

(b) $4 \times 1 = 4$ Turn to page 36

(c) $1 \times 9 = 9$ Turn to page 21

No! The equivalent form needed was 4/4.

Remember we are seeking the equivalent form in order to reduce the fraction.

Try this one.

To reduce $\frac{8 \times 7}{7}$ we must first:

(a) Change it to $8 \times 7/7$ Turn to page 30

(b) Change it to $8/7 \times 7$ Turn to page 27

Incorrect!

We are trying to reduce problems such as $\frac{4 \times 3}{3}$.
To do this we apply what we know about equivalent forms
(such as 3/3 = 1, 2/2 = 1).

With this idea in mind try this one.

To reduce $\frac{8 \times 7}{7}$ we must first:

(a) Change it to 8 x 7/7 Turn to page 30

(b) Change it to 8/7 x 7 Turn to page 27

Your answer was (b) Substitute 1 for 7/7 and multiply by 8. Very good!

The fraction $\frac{6 \times 5}{5}$ reduces to:

(a) 6 Turn to page 37

(b) 5 Turn to page 22

Your answer was (b) Change it to 4 x 3/3. Very good!

Now let's finish the problem.

We want to reduce 4 x 3/3. What should be our next step?

(a)	Multiply	Turn to page 42
(b)	Substitute 1 for 4	Turn to page 45
(c)	Substitute 1 for 3/3	Turn to page 32
(d)	Substitute 3 for 3/3	Turn to page 42

Although it is true that we can change the fraction to 8/7 x 7, this will not help us simplify the fraction.

If we want to simplify a fraction, the procedure we use is to substitute an equivalent form. In the preceding problem 8/7 is not a simple equivalent form but 7/7 is.

Try another.

We want to reduce the fraction $\frac{9 \times 8}{8}$. The equivalent form to look for is:

(a) 9/8 Turn to page 28

(b) 8/8 Turn to page 31

You seem to be having trouble recognizing equivalent forms. Equivalent forms are necessary to be able to reduce fractions.

Return to page 1 and review equivalent forms.

Your answer is incorrect!

You are having your ups and downs. Return to page 93
and begin again. Carefully follow the example and
then continue with the questions.

Good! We need to first change it to 8 x 7/7.

Now that we have 8 x 7/7, what must be done?

(a) Substitute 7 for 7/7 and multiply by 8
Turn to page 34

(b) Substitute 1 for 7/7 and multiply by 8
Turn to page 25

(c) Substitute 1 for 7/7 and multiply by 7
Turn to page 34

Your answer was 8/8. Good, now you're getting back on the track.

Now that you have identified the equivalent form needed we can substitute for it.

The next step in reducing the fraction $\frac{9 \times 8}{8}$ will be:

(a) Substitute 1 for 8/8, and obtain 9 x 1 = 9
Turn to page 33

(b) Substitute 8 for 8/8, and obtain 9 x 8 = 72
Turn to page 41

(c) Substitute 1 for 9/9, and obtain 8 x 1 = 8
Turn to page 44

Your answer was (c) Substitute 1 for 3/3. Excellent! Keep it up.

In reducing $\frac{4 \times 3}{3}$ we have now changed it to 4 x 3/3.

By substituting 1 for 3/3, we have 4 x 1 left. Multiply and we have 4 as our answer.

Now try this one.

In reducing $\frac{5 \times 4}{4}$ we should find the equivalent form for which term?

(a) 5/4 Turn to page 23

(b) 5 x 4 Turn to page 23

(c) 4/4 Turn to page 38

Correct! Substitute 1 for 8/8, and obtain 9 x 1 = 9.

Try one more.

If we reduce the fraction $\frac{5 \times 4}{4}$ we obtain:

(a)	4	Turn to page 19
(b)	5	Turn to page 35
(c)	Neither of the above	Turn to page 19

Incorrect! You should have answered -- Substitute 1 for 7/7 and multiply by 8.

Turn to page 28.

Your answer was 5. Terrific!

Question:

The fraction $\frac{6 \times 5}{5}$ reduces to _____?

(a) 6 Turn to page 37

(b) 5 Turn to page 22

Your answer was (b) 4 x 1 = 4. Very good! I knew you had just made a careless mistake.

The fraction $\frac{2 \times 7}{7}$ reduces to _____?

(a) 7 Turn to page 39

(b) 14 Turn to page 40

(c) Neither of the above Turn to page 43

Correct, it reduces to 6.

You have the right idea on reducing fractions. Now, turn to page 48 and keep up the good work.

Your answer was (c) 4/4. Great!

In reducing $\frac{5 \times 4}{4}$, you first found the equivalent form for 4/4 and it is 1.

Completing the problem, the answer is _____?

(a) 5 Turn to page 35

(b) 4 Turn to page 47

(c) 1 Turn to page 47

Your answer was (a) 7. This is incorrect!

$\frac{2 \times 7}{7}$ is $2 \times 1 = 2$.

You seem to be having your ups and downs. Return to page 20 and carefully follow the examples, then continue from there.

Your answer was (b) 14. Incorrect.

$\frac{2 \times 7}{7}$ is 2 x 7/7 and 2 x 1 = 2

You seem to be having your ups and downs. Return to page 20 and carefully follow the examples and then continue from there.

Your answer was (b) Substitute 8 for 8/8, and obtain 9 x 8 = 72.

Whoa! How can we substitute 8 for 8/8 when 8/8 = 1?

We are substituting equivalent forms so that we can reduce the fractions. Remember equivalent forms are such things as 2/2 = 1, 5/5 = 1, 9/9 = 1.

Try this one.

To reduce $\frac{8 \times 7}{7}$ we must first:

(a) Change it to 8 x 7/7 Turn to page 30

(b) Change it to 8/7 x 7 Turn to page 46

Your answer was (a) Multiply or (d) Substitute 3 for 3/3. Incorrect!

The next step was to substitute 1 for 3/3. Thus, giving us 4 x 1 = 4.

Try this one.

To reduce $\frac{8 \times 7}{7}$ we must first:

(a) Change it to 8 x 7/7 Turn to page 30

(b) Change it to 8/7 x 7 Turn to page 27

You answered (c) Neither of the above. Very good!

You saw that the reduction fo $\frac{2 \times 7}{7}$ was equal to 2.

Turn to page 48.

Your answer was Substitite 1 for 9/9, and obtain

8 x 1 = 8. Incorrect!

Although it is true that 9/9 = 1, there isn't 9/9 in the fraction $\frac{9 \times 8}{8}$. The correct way would be 9 x 8/8 which is 9 x 1 = 9.

Try another.

To reduce $\frac{8 \times 7}{7}$ we must first:

(a) Change it to 8 x 7/7 Turn to page 30

(b) Change it to 8/7 x 7 Turn to page 46

Your answer was (b) Substitute 1 for 4. The correct response should have been to substitute 1 for 3/3, thus giving us 4 x 1 = 4.

Try this one.

To reduce $\frac{8 \times 7}{7}$ we must first:

(a) Change it to 8 x 7/7 Turn to page 30

(b) Change it to 8/7 x 7 Turn to page 27

You answered (b) Change it to 8/7 x 7. Incorrect!

You seem to be having trouble recognizing equivalent forms. Return to page 1 of this unit and review the idea of equivalent forms.

Oops!

You made a careless mistake. Return to page 38 and read it again.

Now that you have mastered reducing fractions using numbers, reducing fractions where the numbers are represented by letters should be easy for you.

For example, if the fraction $\frac{a \times b}{b}$ is to be reduced we treat it exactly like we did with numbers. $\frac{a \times b}{b}$

can be rearranged to read a x b/b and since b/b = 1, we have a x 1 = a.

Note that we did the same thing with the letters as we did with the numbers. We found an equivalent form and substituted it and multiplied to obtain our answer.

Turn to page 49.

If we want to reduce the fraction $\frac{b \times c}{c}$, the first step is to:

(a) Change to b/c x c Turn to page 56

(b) Change to c x b x c Turn to page 51

(c) Change to b x c/c Turn to page 53

Correct! You should substitute 1 for c/c, and obtain

$b \times 1 = b$. Therefore, $\frac{b \times c}{c}$ reduces to b.

Try this one.

The fraction $\frac{t \times r}{r}$ reduces to:

(a) r Turn to page 71

(b) t Turn to page 55

(c) 1 Turn to page 74

Your answer was (b) c x b x c. Wait a minute!

Where did you get an idea like this?

Return to page 48 and read carefully.

You probably misread the answers.

If you rearranged the fraction to read b x c/c, the only substitution you can make is to:

(a) Substitute 1 for c/c, and obtain b x 1 = b
Turn to page 50

(b) Substitute c for c/c, and obtain c x b = 1
Turn to page 70

Your answer was (c) Change it to b x c/c. Good!

You have now changed $\frac{b \times c}{c}$ to b x c/c. Your next step will be to:

(a) Substitute 1 for c/c, and obtain b x 1 = b
Turn to page 50

(b) Substitute 1 for b, and obtain c x 1 = c
Turn to page 52

(c) Substitute c for c/c, and obtain c x b = 1
Turn to page 52

Very good! a is the correct answer.

You saw that $\frac{a \times b}{a}$ is a x b/b which becomes a x 1 = a.

Let's try one more to make sure you have the right idea.

The fraction $\frac{t \times r}{r}$ reduces to:

(a) t — Turn to page 55

(b) r — Turn to page 60

(c) Doesn't reduce — Turn to page 63

Your answer was t. Very good!

The fraction $\frac{E \times I}{I}$ reduces to:

(a) I Turn to page 72

(b) E Turn to page 66

Your answer was (a) Change to b/c x c. Incorrect!

Remember you are looking for an equivalent form so that the fraction can be reduced. b/c is not readily reduced. The equivalent form you should have found was b x c/c. Here c/c can be replaced by 1.

Turn to page 58.

Your answer was (b) Change it to d/t x t. Incorrect!

Possibly a review of equivalent forms will help you.

Return to page 20.

In order to reduce $\frac{d \times t}{t}$, the first step is to:

(a) Change it to d x t/t Turn to page 62

(b) Change it to d/t x t Turn to page 57

Your answer was incorrect!

To reduce d x t/t, you should substitute 1 for t/t, and obtain d x 1 = d.

The only difference between this and say, for example $\frac{3 \times 5}{5}$, is that you are working with letters. With $\frac{3 \times 5}{5}$ you found it to be 3 x 5/5, 5/5 being the fraction for which we can find the equivalent form.

With this in mind reduce the fraction $\frac{a \times b}{b}$. The answer will be:

(a) a Turn to page 54

(b) b Turn to page 61

Your answer was R. Incorrect!

Don't let the difference in the letters fool you. Our equivalent forms relationships still hold. We can say just as easily that r/r = 1 as R/R = 1.

Try another one.

Reducing the fraction $\frac{R \times a}{a}$ becomes:

(a) R — Turn to page 65

(b) a — Turn to page 67

(c) Doesn't reduce — Turn to page 67

The correct answer was a.

You seem to be having trouble with the reducing of fractions.

Tell your teacher where you are and maybe she or he can help you. Then return to page 48.

Good! Your answer was (a) change it to d x t/t.

You have now changed $\frac{d \times t}{t}$ to d x t/t. Your next step will be to:

(a) Substitute 1 for t/t, and obtain d x 1 = d
Turn to page 64

(b) Substitute 1 for d, and obtain t x 1 = t
Turn to page 59

(c) Substitute t for t/t, and obtain d x t = 1
Turn to page 59

No! The fraction will reduce. Let's see why.

Any problem like $\frac{t \times R}{R}$ can be rearranged to read

t x R/R. Now R/R is equal to 1. Therefore, we

can replace R/R by 1. This gives us t x 1 = t.

Try this one.

The fraction $\frac{R \times M}{M}$ reduces to:

(a)	M	Turn to page 69
(b)	R	Turn to page 65
(c)	Doesn't reduce	Turn to page 69

Very good! You chose to substitute 1 for t/t, and obtain

d x 1 = d.

Try another one.

The fraction $\frac{t \times R}{R}$ reduces to:

(a) R — Turn to page 60

(b) Doesn't reduce — Turn to page 63

(c) t — Turn to page 55

Your answer was R. Very good! You weren't fooled.

Keep it up.

Try this one.

The fraction $\frac{t \times B}{B}$ reduces to:

(a)	B	Turn to page 68
(b)	t	Turn to page 55
(c)	Doesn't reduce	Turn to page 68

Yes, the correct answer is E. Very good!

You have done excellently. You didn't let the replacement of numbers by letters throw you.

Now, turn to page 82 and let's find out an easier way to reduce fractions.

Your answer was incorrect!

You seem to be having trouble reducing fractions.

Return to page 20 and do the reducing of fractions using numbers again.

Your answer was incorrect!

Ask your teacher for help and then return to page 48 of this unit.

You have missed the main idea!

Get help from your teacher and then return to page 20.

We cannot substitute c for c/c unless c = 1.

Seek help from your teacher and then return to page 20 and review reducing fractions using numbers.

Your answer of r is incorrect! Let's see why.

We started with the fraction $\frac{t \times r}{r}$. To reduce this fraction, we can change it to t x r/r. Now, r/r = 1, so t x 1 = t.

Try this one.

The fraction $\frac{m \times f}{f}$ reduces to:

(a) f Turn to page 76

(b) m Turn to page 77

Whoa! If you have come this far you shouldn't let this problem throw you.

Let's look at the problem again.

$\frac{E \times I}{I}$ changes to E x I/I. Since I/I = 1 we have

E x 1 = E. Just careless, huh?

Don't be fooled by the capital letters.

Try this one.

$\frac{R \times E}{E}$ reduces to:

(a) R Turn to page 75

(b) E Turn to page 73

Incorrect! What happened to you?

Ask your teacher for help and then return to page 48 of this unit.

Your answer of 1 is incorrect! Let's see why.

We started with the fraction $\frac{t \times r}{r}$. To reduce this we can change it to t x r/r. Now r/r = 1, so t x 1 = t.

Try this one.

The fraction $\frac{m \times f}{f}$ reduces to:

(a) f Turn to page 76

(b) m Turn to page 77

Good! I knew you could do it.

Let's try one more to make sure.

The fraction $\frac{I \times R}{E}$ reduces to:

(a) I — Turn to page 79

(b) R — Turn to page 81

(c) Doesn't reduce — Turn to page 78

Your answer of f is wrong!

Seek help from your teacher and then return to page 48 of this unit.

Your answer was m. Very good! Now you're getting back on the right track.

Let's try one more.

Reduce the fraction $\frac{R \times h}{h}$. It reduces to:

(a) h — Turn to page 68

(b) R — Turn to page 65

(c) Doesn't reduce — Turn to page 63

Very good! It doesn't reduce.

Now, turn to page 82 and let's find out an easier way to reduce fractions.

No! The fraction doesn't reduce to I.

Actually the fraction $\frac{I \times R}{E}$ doesn't reduce at all.

We don't have an equivalent form to substitute into the fraction in order to reduce it.

You had better return to page 48 and continue working from there.

Your answer is incorrect!

See your human teacher and then return to page 93 and read the examples carefully.

No! The fraction doesn't reduce to R.

Actually the fraction $\frac{I \times R}{E}$ doesn't reduce at all.

We don't have an equivalent form to substitute into the fraction in order to reduce it.

You had better return to page 48 and continue working from there.

Hi again!

An easier way to reduce fractions? Sounds pretty good, doesn't it. Actually it involves the same ideas we have been using all along. However, it involves less work and will allow us to reduce fractions of greater size than $\frac{3 \times 2}{2}$ like we have been using.

Let's find out how it's done.

Turn to page 83.

When we had a problem like $\frac{3 \times 2}{2}$ we rearranged it to read $3 \times 2/2$. Then we said that since $2/2 = 1$, we could substitute 1 for $2/2$, thus giving us $3 \times 1 = 3$. Now, rather then rearranging the fraction to read $3 \times 2/2$, we will just write $\frac{3 \times 2}{2}$ and "cancel" out the 2's since they equal one. Thus $\frac{3 \times \cancel{2}}{\cancel{2}} = 3$.

Another example would be $\frac{7 \times \cancel{5}}{\cancel{5}} = 7$.

The main idea is that the numbers we cancel must equal one as they have previously.

Turn to page 84.

You try one.

Reduce the fraction $\frac{3 \times 7}{7}$. The result is:

(a) $\frac{3 \times 7}{7} = 7$ Turn to page 89

(b) $\frac{3 \times 7}{7} = 3$ Turn to page 88

Your answer was $\frac{7 \times 11}{7} = 7$. This is incorrect!

Perhaps the difference in order fouled you up. When we are multiplying the order of the numbers has no effect on the answer.

For example, $\frac{2 \times 3}{2}$ is the same as $\frac{3 \times 2}{2}$. It doesn't make any difference whether we write

2/2 x 3 = 3 or 3 x 2/2 = 3.

Therefore, the above problem should be 7/7 x 11 = 11 or in cancellation form $\frac{7 \times 11}{7} = 11$.

Turn to page 86.

Now try this one.

Reduce by cancellation the following fraction:

$\frac{8 \times 3}{8}$.

(a) $\frac{\cancel{8} \times 3}{\cancel{8}} = 3$ Turn to page 96

(b) $\frac{8 \times \cancel{3}}{\cancel{8}} = 8$ Turn to page 99

(c) $\frac{\cancel{8} \times 3}{\cancel{8}} = 8$ Turn to page 100

Your answer was $\frac{7 \times 5}{5} = 7$. Very good! Now you're getting it.

Reduce this fraction by cancellation: $\frac{7 \times 11}{7}$

(a) $\frac{7 \times 11}{7} = 11$ Turn to page 90

(b) $\frac{7 \times 11}{7} = 7$ Turn to page 85

(c) Not sure Turn to page 91

Very good! $\frac{3 \times 7}{7}$ cancels to 3.

What about $\frac{7 \times 11}{7}$?

(a) $\frac{7 \times 11}{7} = 11$ Turn to page 90

(b) $\frac{7 \times 11}{7} = 7$ Turn to page 102

(c) Not sure Turn to page 97

Your answer was (a) $\frac{3 \times 7}{7} = 7$. Incorrect!

Let's take another look. If we were going to reduce this fraction the way we have previously, we would write 3 x 7/7 and since 7/7 = 1 we would have 3 x 1 = 3.

Therefore, if we want to cancel we must cancel the 7/7. The fraction would look like $\frac{3 \times 7}{7} = 3$ when reduced.

Try another one.

Reduce the fraction $\frac{7 \times 5}{5}$ by cancellation.

(a) $\frac{7 \times 5}{5} = 7$ Turn to page 87

(b) $\frac{7 \times 5}{5} = 5$ Turn to page 95

Excellent! $\frac{7 \times 11}{7}$ is 11.

Apparently you know that when we multiply, the order doesn't matter. For example, $\frac{11 \times 7}{7}$ is the same as $\frac{7 \times 11}{7}$.

Turn to page 93.

Your answer was (c) not sure. This is incorrect.

Perhaps the difference in order fouled you up. When we are multiplying the order of the numbers has no effect on the answer.

For example, $\frac{2 \times 3}{2}$ is the same as $\frac{3 \times 2}{2}$. It doesn't make any difference whether we write $2/2 \times 3 = 3$ or $3 \times 2/2 = 3$.

Therefore, the above problem should be $7/7 \times 11 = 11$ or in cancellation form $\frac{\cancel{7} \times 11}{\cancel{7}} = 11$

Turn to page 86.

Excellent! $\frac{3 \cdot 7 \cdot \cancel{2}}{\cancel{2} \cdot 5} = \frac{21}{5}$ is correct.

You noticed that the only numbers we could cancel were the two's because 2/2 = 1.

To what does the fraction $\frac{5 \cdot 2 \cdot 7 \cdot 3}{3 \cdot 5 \cdot 2 \cdot 7 \cdot 3}$ reduce?

(a) 1/3 Turn to page 98

(b) 1 Turn to page 104

(c) 35/15 Turn to page 103

We can now use the "cancellation" idea to reduce a string of numbers.

For example, $\frac{2 \cdot 5 \cdot 3}{3 \cdot 5}$ can be cancelled to

look like $\frac{2 \cdot 5 \cdot \cancel{3}}{\cancel{3} \cdot 5}$ (Cancelling the 3's)

$\frac{2 \cdot \cancel{5}}{\cancel{5}}$ (Cancelling the 5's)

which leaves us with 2.

We could have written it 2 x 5/5 x 3/3 which becomes 2 x 1 x 1 = 2. However, by using cancellation we save time.

Turn to page 94.

Remember, you can only cancel when the number is the same in both the numerator and the denominator.

You try this one.

Reducing the fraction $\frac{3 \cdot 7 \cdot 2}{2 \cdot 5}$ by cancelling is:

(a) $\frac{\cancel{3} \cdot 7 \cdot 2}{\cancel{2} \cdot 5} = \frac{14}{5}$ Turn to page 103

(b) $\frac{3 \cdot 7 \cdot \cancel{2}}{\cancel{2} \cdot 5} = \frac{21}{5}$ Turn to page 92

(c) $\frac{3 \cdot \cancel{7} \cdot 2}{2 \cdot \cancel{5}} = \frac{6}{2}$ Turn to page 103

Your answer was incorrect!

Ask your human teacher for help and then return to page 83 and study the example carefully.

Your answer was (a) $\frac{8 \times 3}{8} = 3$. Very good!

Try this one.

$\frac{3 \times 13}{10}$ reduces to:

(a) 39 — Turn to page 95

(b) $\frac{3 \times 13}{10} = 9$ — Turn to page 99

(c) Doesn't cancel or reduce — Turn to page 101

Your were not sure. Let's take a look.

We have the fraction $\frac{7 \times 11}{7}$. It doesn't make any difference whether we have $\frac{7 \times 11}{7}$ or $\frac{11 \times 7}{7}$ since multiplication has no order. We can multiply 11 x 7 just the same as 7 x 11.

Because this is true, 7/7 x 11 is 1 x 11 or 11.

Therefore, you should have written $\frac{7 \times 11}{7} = 11$.

Turn to page 86 and continue.

Very good! $\frac{5 \cdot 2 \cdot 7 \cdot 3}{3 \cdot 5 \cdot 2 \cdot 7 \cdot 3}$ reduces to 1/3.

That ends this section of instruction.

Turn to page 113 (Book II) and we'll discuss the nature of the integers we have been discussing.

Your answer was incorrect!

Ask your human teacher for help and then return to page 83 and carefully study the example.

Your answer was incorrect!

Ask your human teacher for help and then return to page 83 and carefully study the example.

Excellent! It doesn't cancel.

Now, let's go on to a different type of fraction.

Turn to page 93.

Your answer was $\frac{7 \times 11}{7} = 7$. Incorrect!

7/7 = 1 not 11/7, so you should have cancelled like this: $\frac{7 \times 11}{7} = 11$.

We could have written 7/7 x 11 or 1 x 11 = 11.

However, since 7/7 = 1 we can just cancel.

Turn to page 86 and continue.

Your answer was incorrect!

Return to page 93 and carefully study the example.

If you still have trouble understanding the process, seek help from your human teacher.

Your answer was (b) 1. Incorrect!

Let's look at the problem again:

$$\frac{5 \cdot 2 \cdot 7 \cdot 3}{3 \cdot 5 \cdot 2 \cdot 7 \cdot 3}$$

We have 5/5 = 1 so they cancel. Also 2/2 = 1 so they cancel, and 7/7 = 1 so they cancel.

We have one 3 left on top and two 3's on the bottom. We can pair the 3 on the top with one of the 3's on the bottom. Thus, one set of 3's cancel and leaves us a 3 on the bottom or $\frac{1 \cdot 3}{3 \cdot 3} = \frac{1}{3}$.

Try this one.

Reduce the fraction $\frac{2 \cdot 5 \cdot 7 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 5 \cdot 7}$ to:

(a) 2 Turn to page 109

(b) 6 Turn to page 112

(c) 2/3 Turn to page 108

11/3 is correct!

Let's try one more.

Reducing $\frac{2 \cdot 2 \cdot 11 \cdot 7 \cdot 2 \cdot 3}{2 \cdot 7 \cdot 3 \cdot 11}$ we obtain:

(a) 3 Turn to page 107

(b) 4 Turn to page 111

(c) 7 Turn to page 107

Your answer is incorrect!

See your human teacher and then return to page 93

and read the examples carefully.

Your answer was incorrect!

You are having your ups and downs. Return to page 93
and start this section over.

Your answer was 2/3. Very good!

Try one more.

Reduce the fraction $\frac{2 \cdot 2 \cdot 11 \cdot 7 \cdot 2 \cdot 3}{2 \cdot 7 \cdot 3 \cdot 11}$

(a) 3 Turn to page 29

(b) 4 Turn to page 111

(c) 7 Turn to page 103

Your answer of 2 is incorrect!

Let's see why.

We have the fraction $\frac{2 \cdot \cancel{5} \cdot \cancel{7} \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot \cancel{5} \cdot \cancel{7}}$

We can cancel the 5's and the 7's.

This leaves us with $\frac{2 \cdot \cancel{3} \cdot \cancel{3}}{3 \cdot \cancel{3} \cdot \cancel{3}}$

Two pairs of 3's will cancel, which leaves us with 2/3.

Try this one.

Reducing the fraction $\frac{11 \cdot 7 \cdot 2 \cdot 3 \cdot 11}{3 \cdot 2 \cdot 7 \cdot 3 \cdot 11}$ our answer is:

(a) 11/3 Turn to page 105

(b) 11 Turn to page 106

(c) 3/11 Turn to page 110

Your answer is incorrect!

See your human teacher and then return to page 93
and read the examples carefully.

4 is the correct answer. Excellent!

That ends this section of instruction.

Turn to page 113 (Book II) and we'll discuss the nature of the integers we have been discussing.

Your answer of 6 is incorrect!

Let's see why.

We have the fraction $\frac{2 \cdot 5 \cdot 7 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 5 \cdot 7}$

We can cancel the 5's and the 7's.

This leaves us with $\frac{2 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3}$

Two pairs of 3's will cancel, which leaves us with 2/3.

Try this one.

Reducing the fraction $\frac{11 \cdot 7 \cdot 2 \cdot 3 \cdot 11}{3 \cdot 2 \cdot 7 \cdot 3 \cdot 11}$ our answer is:

(a) 11/3 Turn to page 105

(b) 11 Turn to page 80

(c) 3/11 Turn to page 110